Sample Questions

Limits

Find the following limits

1. $\lim_{x \to -3} (-2x^2 + 5x + 8)$

2. $\lim_{\theta \to 0} \frac{\sec\theta - 1}{\theta \sec\theta}$

3. $\lim_{x \to -2} \frac{|x+2|}{x+2}$

4. $\lim_{x \to 3} \frac{x-3}{(x^3 - 27)}$

5. Describe the concept of a limit of a specific function both analytically (table) and graphically

Continuity

Find the limits (if it exists) and if not explain why.

1. $\lim_{x \to -3^-} \frac{x}{\sqrt{x^2 - 9}}$

2. $\lim_{x \to \pi} \cot x$

Find the x-values where f is not continuous

3. $f(x) = \frac{x}{x^2 - x}$

4. $f(x) = \frac{|x+2|}{x+2}$

3 Explain why. $f(x) = x^3 + 3x - 2$ has a zero in $[0, 1]$.

Sample Question Key

Limits

1. $\lim_{x \to -3} (-2x^2 + 5x + 18) = -2(-3)^2 + 5(-3) + 18 = \boxed{-15}$

2. $\lim_{\theta \to 0} \frac{\sec\theta - 1}{\theta \sec\theta} = \frac{\frac{1}{\cos\theta} - 1}{\theta \frac{1}{\cos\theta}} \cdot \frac{\cos\theta}{1} = \frac{1 - \cos\theta}{\theta} = 0$

3. $\lim_{x \to -2} \frac{|x+2|}{x+2}$

-1.99	-1.999	-2	-2.001	-2.01
1	1	DNE	-1	-1

the limit does not exist because the lines don't connect.

4. $\lim_{x \to 3} \frac{x-3}{(x^3-27)} = \frac{x-3}{(x-3)(x^2+3x+9)} = \frac{1}{27}$

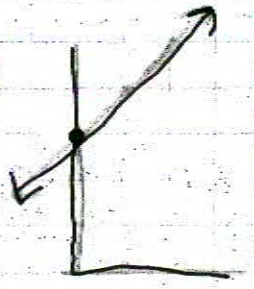
to factor →

3	1	0	0	-27
		3	9	27
	1	3	9	0

5. A limit is what is happening to the y value as x approaches a critical number.

answer will vary

$\lim_{x \to 2} x + 3$



1.9	1.99	1.999	2	2.001	2.01	2.1
4.9	4.99	4.999	5	5.001	5.01	5.1

Continuity

1. $\lim_{x \to -3^-} \frac{x}{\sqrt{x^3 - x}}$ best way to solve is to graph. Limit does not exist. The function decreases without bound as x approaches -3 from the left

2. $\lim_{x \to \pi} \cot(x)$ The limit does not exist. The function decreases without bound as x approaches π from the left and increases without bound as x approaches π from the right.

3. $f(x) = \frac{x}{x^2 - x} = \frac{x}{x(x-1)}$ it is not continuous at $x = 0$ and $x = 1$

4. $f(x) = \frac{|x+2|}{x+2}$ not continuous at $x = -2$

5. $f(0) = -2$
$f(1) = 2$

$f(x)$ is continuous from $[0, 1]$ because of all of this and because $f(0)$ is negative and $f(1)$ is positive thus according to IVT $f(x)$ must cross the x-axis.

Practice Problems

Graphs

1.) Find the slope and y-intercept of $6x - 5y = 15$

2.) Find the equation of the line that passes through $(2, 8)$ and $(5, 0)$

Assymptotes

1.) Find all vertical asymptotes of $f(x)$ if $f(x) = \frac{3x-6}{(x-2)(2x^2+x-1)}$

2.) Find all asymptotes of $f(x)$ if $f(x) = \frac{6x^2-2x}{5x-9x^3}$

Answer Key

1.) Find slope and y-intercept of $6x - 5y = 15$

$6x - 5y = 15$

$-5y = 15 - 6x$

$y = \frac{6}{5}x - 3$

Slope $= \frac{6}{5}$

y-intercept at -3

2.) Find the equation of the line that passes through $(2, 8)$ and $(5, 0)$

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 0}{2 - 5} = -\frac{8}{3}$ $\quad (5, 0)$

$y = -\frac{8}{3}x + \frac{40}{3}$

1.) Find all vertical assymptotes of $f(x) = \frac{3x - 6}{(x-2)(2x^2 + x - 1)}$

at $x = 2$ $\quad f(x) = \frac{0}{0}$ so no assymptote

at $x = 0.5$ $\quad f(x) = \frac{-4.5}{0}$ so V.A. at $x = 0.5$

at $x = -1$ $\quad f(x) = \frac{-9}{0}$ so V.A. at $x = -1$

2.) Find all asymptotes of $f(x) = \frac{6x^2 - 2x}{5x - 9x^3}$

$\frac{6x^2 - 2x}{(5x - 9x^3)} = \frac{6x^2 - 2x}{(x)(5 - 9x^2)}$

at $x = 0$ $\quad f(x) = \frac{0}{0}$ so no asymptote

$5 - 9x^2 = 0$

$x^2 = \frac{5}{9}$

$x = \pm\frac{\sqrt{5}}{3}$

So H.A. at $y = 0$

V.A. at $x = \pm\frac{\sqrt{5}}{3}$